Simulation of Batch Sedimentation with Compression

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The currently accepted theory of sedimentation is the theory of Kynch, developed in 1952. It establishes the existence of waves of equal concentration that propagate with constant speed through the suspension. This theory predicts that within the suspension, the curves of equal concentration in the z-t plane are straight lines (Kynch, 1952; Wallis, 1962; Shannon et al., 1964; Concha and Bustos, 1986). Unfortunately, the sparse published experimental evidence on the distribution of concentration during batch sedimentation of flocculate suspension (Scott, 1968; Been and Sills, 1981) contradicts this model. Actually, the experimental data show that the lines of equal concentration, for the higher concentration range, are curves with decreasing slope that become horizontal as time increases.

Obviously, the predictions of the Kynch theory or any of its recent modifications (Tiller, 1981; Fitch, 1983; Concha and Bustos, 1986), violate these experimental results. Instead of continuing with this kinematical model, the study of a more complete model (Concha and Bustos, 1985) that would include dynamical effects is in order. It is the purpose of this work to analyze the numerical solution of a dynamic model for batch sedimentation of flocculated suspensions and to show the influence of the parameters of the solid pressure on the shape of the lines of equal concentration.

Dynamical Model

Consider a two-phase solid-water mixture. Let $\phi\ (z, t)$ and $v_s(z, t)$ be the volume fraction and the velocity, respectively, of the solid component. Defining the solid flux density by $f = \phi\, v_s$, the following one-dimensional conservation equations describe the sedimentation of the suspension under gravity (Concha and Bustos, 1985):

$$\frac{\partial \phi}{\partial t} + \frac{\partial f}{\partial z} = 0 \tag{1}$$

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial z}\left[\frac{f^2}{\phi} + \frac{p_s(\phi)}{\rho_s}\right] = -\frac{\Delta\rho g \phi}{\rho_s} + \frac{\Delta\rho g f}{\rho_s U(\phi)} \tag{2}$$

in the domain $\Omega = [(z, t)\,|\,0 \leq z \leq L,\ 0 \leq t]$, where L is the initial height of the mixture, ρs the solid density, $\Delta\rho$ the solid-fluid density difference, and g the acceleration of gravity. The two functions $U(\phi)$ and $p_s(\phi)$ represent the free settling velocity of the suspension and the pressure in the solid component, respectively. These functions are given by

$$U(\phi) = u_\infty (1 - \phi)^{\alpha+1} \tag{3}$$

$$p_s(\phi) = a\, e^{b\phi} \tag{4}$$

where u_∞, α, a, and b are experimental parameters.

Equation 1 is the continuity equation for the solid component in the mixture and Eq. 2 is its linear momentum balance. In this last equation, the first term is the rate of change of the linear momentum, the second term is the nonlinear convective momentum transfer, and the third term represents the solid pressure gradient of the suspension. On the righthand side of Eq. 2, the first term stands for the external forces discounting buoyancy and the last term is the hydrodynamic interaction force between the solid and the fluid component in the mixture. In what follows we will neglect the convective term since it is several orders of magnitude smaller than the other terms.

The initial and boundary conditions are given by

$$\phi(z, 0) = \phi_o, \quad 0 < z \tag{5}$$

$$f(z, 0) = \phi_o U(\phi_o), \quad 0 < z \tag{6}$$

$$f(0, t) = 0, \quad 0 < t \tag{7}$$

Equations 1 and 2 constitute a nonlinear hyperbolic system, and its solutions are in general discontinuous. On the points of discontinuity $z(t)$, the Rankine-Hugoniot, or jump conditions for the mass and linear momentum, *are satisfied:*

$$\sigma = \frac{f^+ - f^+}{\phi^+ - \phi^-} = \frac{p^+ - p^-}{f^+ - f^-} \tag{8}$$

Here $\sigma = dz(t)/dt$ is the speed of propagation of the discontinuity and $\phi\pm = \phi[z(t) \pm 0, t]$. The characteristic roots λ_1 and λ_2 of the system, also known as characteristic speeds, are given by

$$\lambda_1 = \frac{f}{\phi} + \sqrt{\frac{1}{\rho_s}\frac{dp_s}{d\phi}}; \quad \lambda_2 = \frac{f}{\phi} - \sqrt{\frac{1}{\rho_s}\frac{dp_s}{d\phi}} \tag{9}$$

A discontinuity $z(t)$ which satisfies the Lax entropy condition

$$\lambda_i(\phi^-) \geq \sigma \geq \lambda_i(\phi^+) \tag{10}$$

for $i = 1$ or 2, is called a shock associated with the characteristic speeds λ_1 or λ_2.

Numerical Solution

To solve the problem comprising Eqs. 1 to 7 numerically we use the following finite-difference scheme. Let the domain Ω be covered by a uniform rectangular grid defined by the lines $t = nk$ and $z = jh$, where h and k are the space and time steps, respectively. The approximating finite-difference scheme to Eqs. 1 and 2 is:

$$\frac{\phi_j^{n+1} - \phi_j^n}{k} + \frac{\frac{3}{2}f_j^n - 2f_{j-1}^n + \frac{1}{2}f_{j-2}^n}{h} = 0, \quad j = 2, \ldots, N \tag{11}$$

$$\frac{\phi_1^{n+1} - \dfrac{\phi_o^n + \phi_2^n}{2}}{k} + \frac{f_2^n - f_o^n}{h} = 0 \tag{12}$$

$$\frac{f_j^{n+1} - \frac{1}{2}(f_{j+1}^n + f_{j-1}^n)}{k} + \frac{p_s(\phi_{j+1}^{n+1}) - p_s(\phi_{j-1}^{n+1})}{2h\rho_s} = -\frac{\Delta\rho g}{\rho_s}\left[\phi_j^{n+1} - \frac{f_j^{n+1}}{U(\phi_j^{n+1})}\right], \quad j = 1, \ldots, N \tag{13}$$

for all $n = 0, 1 \ldots$ and $N = L/h$.

Equations 11 and 12 are respectively the three-point upwind and the Lax-Friedrich scheme to solve Eq. 1 (Bustos, 1984), and Eq. 13 is an implicit-difference scheme to solve Eq. 2.

Simulation

To simulate the settling behavior of compressible suspensions, we make use of experimental results on flocculated tailings from the copper concentrator of the Chuquicamata Division of Codelco Chile. These are the same data used in our previous work (Concha and Bustos, 1986). The experimental parameters were determined by standard sedimentation and uniaxial consolidation tests by Becker (1982); they are listed in Table 1.

Table 1. Experimental Parameters

$u_\infty = -6.05 \times 10^{-4}$ m/s
$\alpha = 11.59$
$a = 5.35$ N/m^2
$b = 17.9$
$L = 0.405$ m
$\phi_0 = 0.123$
$\rho_s = 2{,}500$ kg/m^3

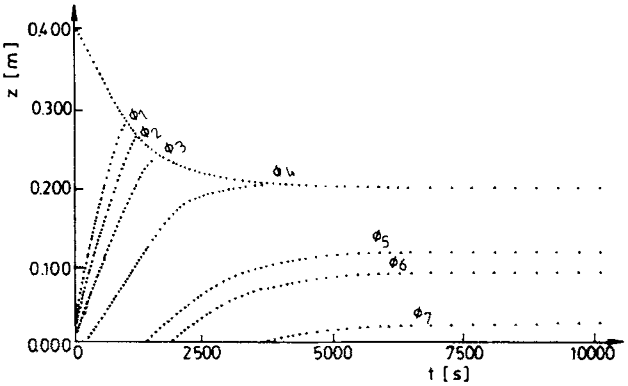

Figure 1. Simulated z vs. t plot for settling of a flocculated suspension, showing lines of equal concentration.

$\phi_1 = 0.13$ $\phi_5 = 0.15$
$\phi_2 = 0.14$ $\phi_6 = 0.28$
$\phi_3 = 0.16$ $\phi_7 = 0.30$
$\phi_4 = 0.20$

Using these parameters, the numerical solution to the problem in Eqs. 1 to 7 is given in Figure 1.

Discussion

In our previous work (Concha and Bustos, 1986), we stressed the fact that within the Kynch theory, the curves of equal concentration were straight lines. We also indicated that the water-suspension interface could be well represented for flocculated suspensions by the modified Kynch theory, but that the concentration profiles do not correspond to the experimental ones.

On the other hand, the simulation obtained by the numerical solution of Eqs. 1 to 7, represented by Figure 1, shows that in this case the lines of equal concentration do have the desired shape and correspond qualitatively to the experimental data of Scott (1968) and Been and Sills (1981). This similarity in the shape of the curves of equal concentration and the good agreement between experimental and simulated values of the water-suspension interface, shown in Concha and Bustos (1985), give strong support to the dynamic model presented in this paper.

The shape and location of the lines of equal concentration depend strongly on the values of the parameters of the solid pressures, for equal initial settling rate. A decrease in the parameters a and b makes the line of equal concentration move closer to the water-suspension interface. This was to be expected since a decrease in a and b means that the suspension is more compressible and therefore, for the same height of sediment, the concentration of a certain fixed layer must be higher.

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Notation

a = parameter, Eq. 4
b = parameter, Eq. 4
f = solid flux density
g = acceleration of gravity constant

h – space step
j – subscript for space variable
k – time step
n – superscript for time variable
N – L/h
L – initial height of mixture
o – subscript for initial concentration
p_s – solid partial pressure
s – subscript for solid constituent
t – time
u_∞ – terminal settling velocity
v_s – velocity of solid constituent
z – vertical space variable

Greek letters

α – exponent, Eq. 3
ϕ – volume fraction of solids
λ_1, λ_2 – characteristic speeds
ρ_s – density of solid constituent
$\Delta\rho$ – solid-fluid density difference
σ – speed of propagation of a discontinuity
Ω – domain

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